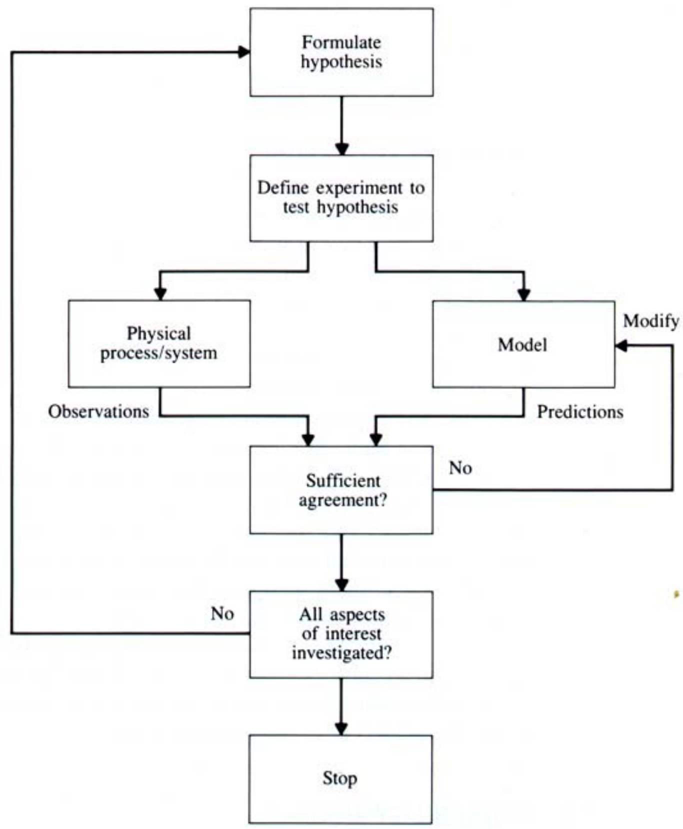
Chapter 1 – Probability Models

Mathematical Models

- **Experiments:** Costly way of testing a design or solve a problem.
- **Model:** Approximate representation of a physical situation.
- **Useful Model:** Able to explain all relevant aspects of a given phenomenon.
- **Mathematical Models:** If observational phenomenon has measurable properties then a mathematical model consisting of a set of assumptions about the system is employed.

Conditions under which an experiment is performed and a model is assumed are very critical. Change the assumptions then a "good" model can be a great fiasco.



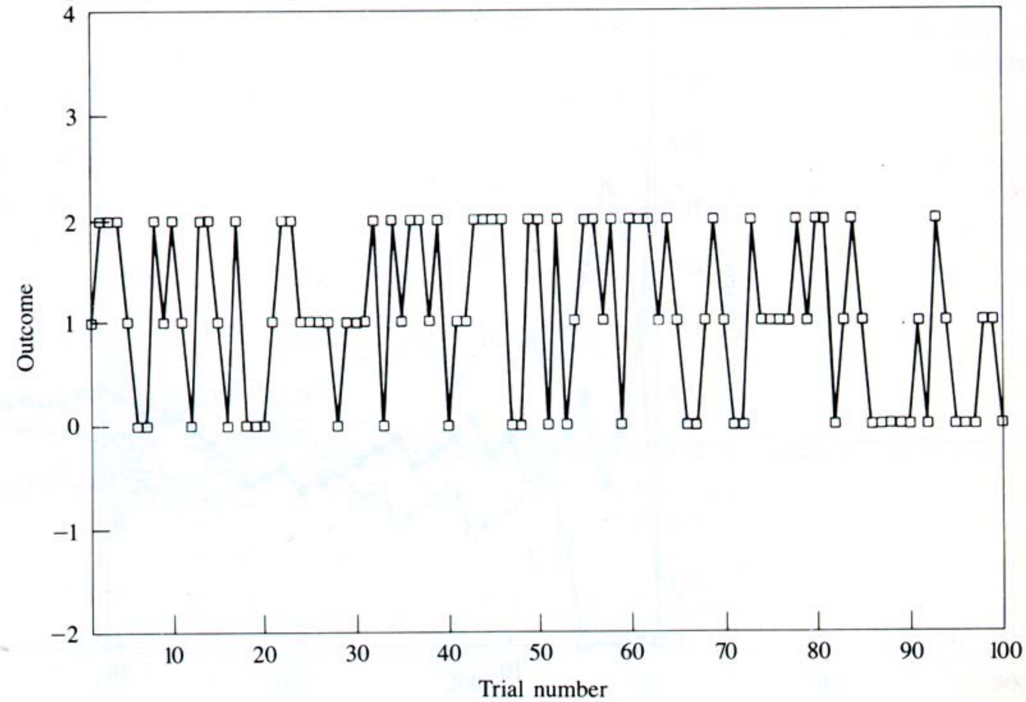
- Computer Simulation Models: They mimic or simulate the dynamics of a system
- Deterministic Models: Lab and textbook cases, conditions determine outcome
    1. Circuit Theory
    2. Ohm's Law
    3. Kirchoffs' Laws
    4. Transforms: FFT; Laplace Transforms
    5. Convolution :Input/output behavior of systems with well-defined coefficients

- Probabilistic (Stochastic, Random) Models: involve phenomena that exhibit unpredictable variation and randomness.

Ex: Urn with three balls; (0,1,2 marked)

-- Outcome: A number from the set {0,1,2}

-- Sample Space: All possible outcomes of an experiment: S = {0,1,2}



Statistical Regularity:

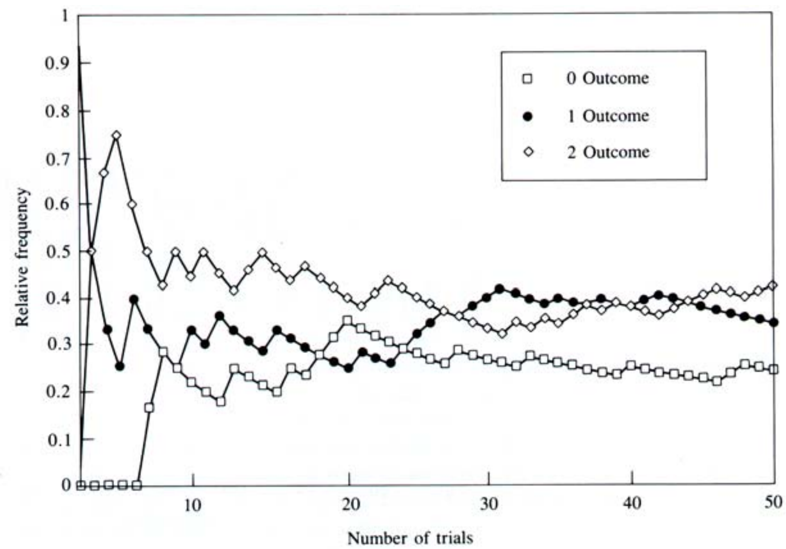
Relative Frequency: $f_k(n) = \dfrac{N_k(n)}{n}$

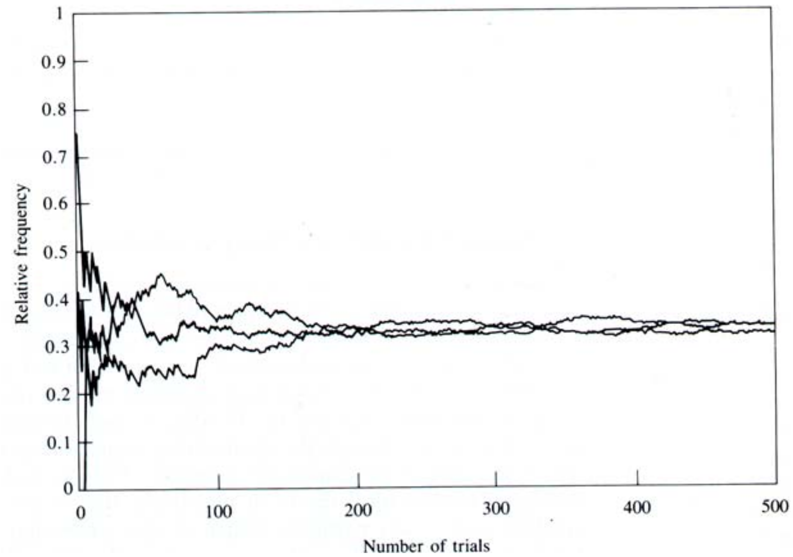
n: # of experiments under identical conditions;

k: Counter index

$$\left.\begin{array}{l} N_0\text{: \# of “0” balls} \\ N_1\text{: \# of “1” balls} \\ N_2\text{: \# of “2” balls} \end{array}\right\} \text{ in } n \text{ tries}$$

Let $n \to \infty$ then $\lim\limits_{n\to\infty} f_k(n) = P_k$; where $\mathbf{P_k}$**:** probability of the outcome $\mathbf{E_k}$.





Properties of Relative Frequency:

Suppose a random experiment has K possible outcomes: S = {1,2,…,K}. Then in "n" trials we have

$$0 \le N_k(n) \le \text{n} \quad \text{for k} = 1,2,\ldots,K \quad \Rightarrow \quad 0 \le f_k(n) \le 1$$

and

$$\sum_{k=1}^{N} N_k(n) = n \qquad \text{which implies:} \qquad \sum_{k=1}^{N} f_k(n) = 1$$

Event ≡ Any outcome of an experiment satisfying certain condition(s).

Ex: Consider the 3-ball urn experiment

A: even = {0,2} then,

$$f_A(n) = \frac{N_0(n) + N_2(n)}{n} = f_0(n) + f_2(n)$$

Disjoint (mutually exlusive) events: If A or B can occur but not both, then

$$f_C(n) = f_A(n) + f_B(n)$$

Relative frequency of two disjoint events is the sum of their individual relative frequency.

Kolmogorov's axioms to form a Theory of Probability: Assumptions:

1. Random experiment has been defined and the sample space S has been identified.
2. A class of subsets of S has been specified.
3. Each event A has been assigned a number P(A) such that,
    1. $0 \le P(A) \le 1$
    2. $P(S) = 1$
    3. If A and B are mutually exclusive events then
       $$P(A \text{ or } B) = P(A) + P(B)$$

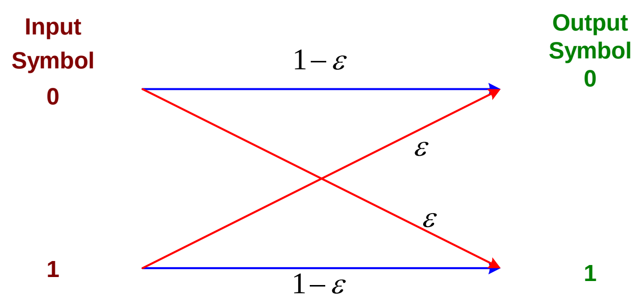
Kolmogorov's axioms are sufficient to build a consistent Theory of Probability.

Example: Packet Voice Communication system Efficiency
Due to silences voice communication is very inefficient on dedicated lines. It is observed that only "1/3" of the time actual speech goes through. How to increase this rate by using prob. approaches???

Solution: Error vs rate trade off in digital information (BCS) transmission/storage

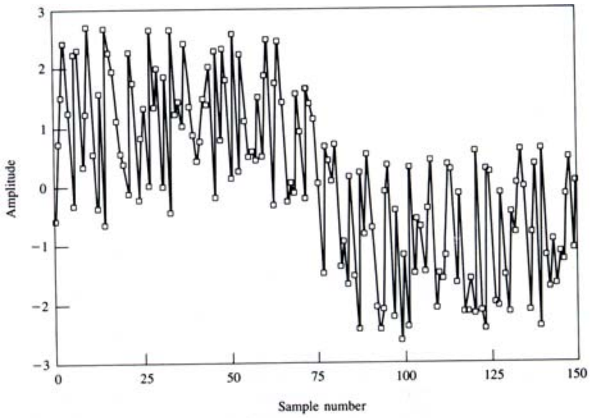
$$\left.\begin{array}{l} 0 \rightarrow 0\ 0\ 0 \\ \\ 1 \rightarrow 1\ 1\ 1 \end{array}\right\} + \text{ Majority Rule to make decision at the receiver.}$$

If each bit has a $P_e = 10^{-3}$ due to this simple scheme, $P_e \rightarrow 3\text{x}10^{-6}$.
Cost: Rate is decrease to 1/3 of the original.
Binary Symmetric Channel with cross-over probability: $\varepsilon = P_e = 10^{-3}$

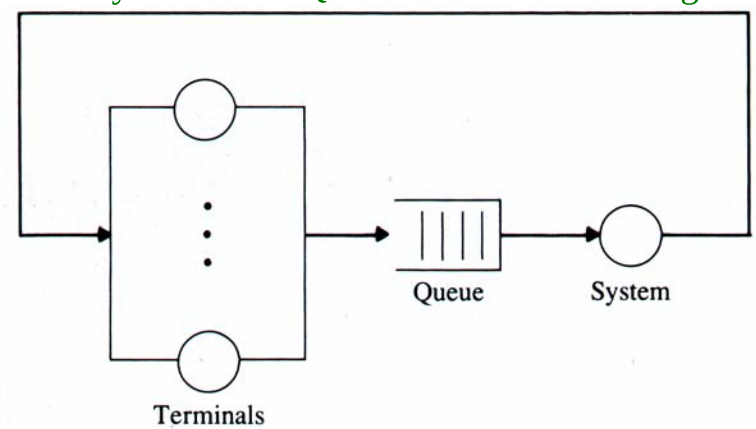


Example: Signal Enhancement Using Filters

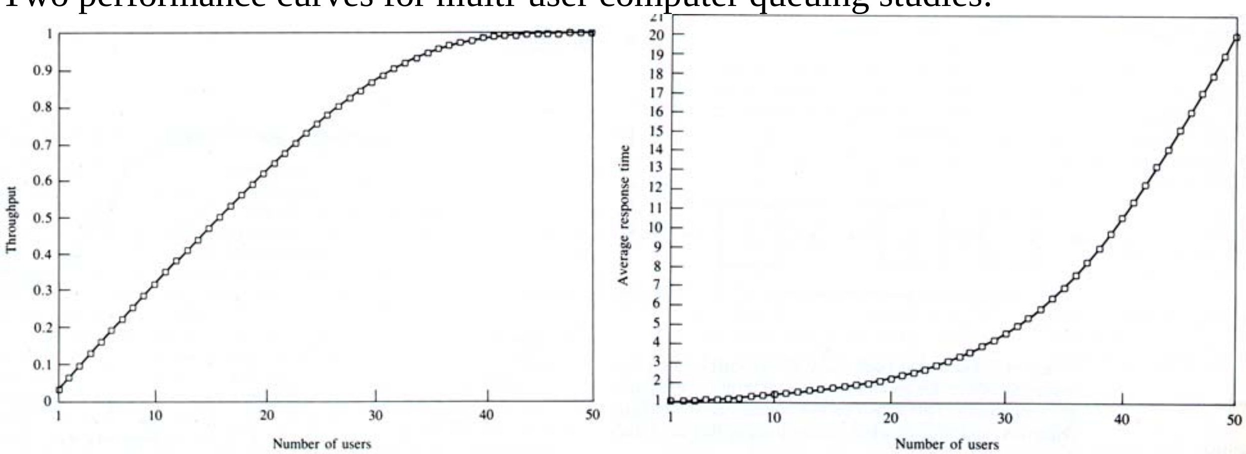
Given a signal x(t) corrupt with noise and has a Signal-to-Noise Ratio value SNR. If you filter this noisy signal with a properly designed –hopefully adaptive, filter to suppress noise, we obtain an enhanced signal, (smoothed by the filter.)



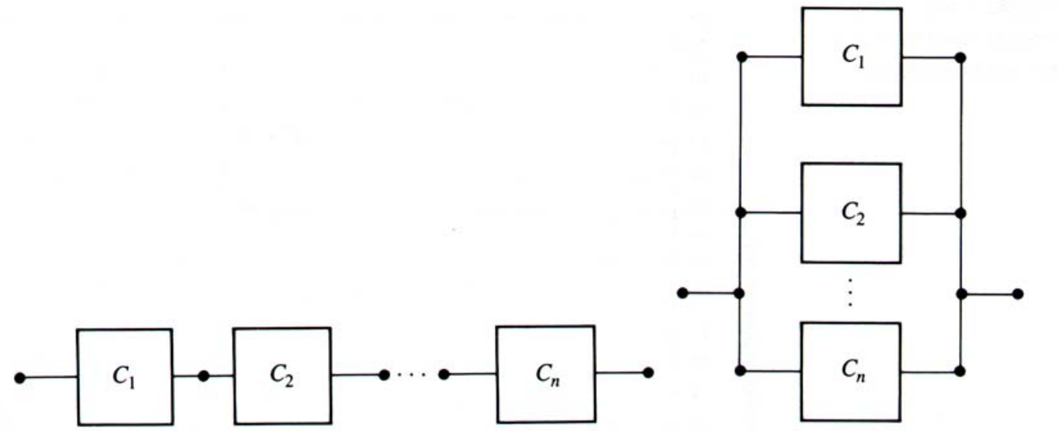
Example: Multi User Systems with Queues: Resource sharing



Two performance curves for multi-user computer queuing studies:



Example: System Reliability: Cascade vs. Parallel Systems



(a) Series configuration of components. (b) Parallel configuration of components.

Issues: Need of a clock vs. the system delay or throughput rate.

Ex: Prob. 1.1

Experiment: Selecting two balls in succession from an urn with 2 black + 1 white ball without replacement.

a) Sample Space: S = {bb, bw, wb} If: 1st ball is B, then 2nd is B or W
If: 1st ball is W, then 2nd is only B

b) With replacement: All outcomes are W or B, 2then
S = {bb, bw, wb, ww}

c) $f_{ww}(n)$ in part (a)?
Not possible to have W then W $\Rightarrow f_{ww}(n) = 0$

$f_{ww}(n)$ in part (b)?

$N_w(n) = 1/3$, since 1 W and 2 B balls always
Of these 1/3 outcomes again 1/3 are W since balls are replaced.

$\therefore f_{ww}(n) = (1/3)*(1/3) = 1/9$

2nd draw is effected by the outcome of 1st draw?
a) Yes
No.